Negative Impedances and the Twin 21-Type Repeater

By GEORGE CRISSON

This paper discusses negative resistances and impedances. It describes their properties and some devices by which they may be produced physically. Certain properties of negative impedances when used as series and shunt boosters for amplifying speech waves in telephone circuits are discussed. The paper concludes with a description of the circuit and properties of the twin 21-type repeater.

WHEN an e.m.f. is applied to the terminals of an ordinary positive resistance a current flows in at the terminal connected to the positive pole of the source and out at the other terminal. This direction of current flow is considered positive and the value of the resistance R, in ohms is given by Ohm's law as $R = E/I$ where E is the applied voltage and I is the current in amperes. Similarly a definite current I may be passed through the resistance and a potential difference or drop $E = RI$ will appear across its terminals. With positive resistances it makes no difference whether we "apply an e.m.f." or "pass a current". The resistance may be a very simple device such as a coil of wire which absorbs energy from the circuit at a rate $W = EI = I^2R$ watts.

It is possible, however, to construct assemblages of apparatus which have the property of keeping the ratio of the voltage across a pair of terminals to the current at the terminals constant, but with the relative direction of the voltage and current opposite to that which a positive resistance would give. In such devices the resistance is negative and the apparatus contributes power to the circuit with which it is connected. Each such device necessarily includes a source of energy such as a battery and some means such as a vacuum tube for controlling the delivery of this energy to the circuit. There are two varieties of such devices. In one case, the internal arrangement of the mechanism is such that, if a definite voltage is applied to the terminals, a current flows in a direction opposite to the applied e.m.f. In the other, if a definite current is passed through the system, the drop across the terminals will be opposite in direction to that caused by a positive resistance. These two arrangements are essentially different and cannot be used interchangeably in a given circuit, though either one can give any desired value of negative resistance. If the wrong arrangement is used instability or singing will occur. To know whether a given negative resistance will work satisfactorily in a given circuit it is not sufficient to know its value in ohms. Something must be known

about its internal arrangement and about the impedance of the circuit in which it is to work.

Regenerative Negative Resistances

One of the simplest ways to produce a negative resistance is to interconnect the input and output terminals of a one-way amplifier. This gives a regenerative arrangement because part of the output energy of the amplifier is fed back into the input circuit. The type of negative resistance obtained depends upon the way in which the interconnection is made.

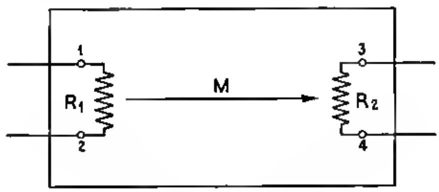

Fig. 1—Ideal one-way amplifier.

Fig. 1 shows schematically an ideal one-way amplifier for this purpose. It has a pair of input terminals 1, 2, and a pair of output terminals 3, 4. The impedances between the input and output terminals are pure resistances R_1 and R_2, respectively. Some mechanism, indicated symbolically by the arrow, is provided, which produces an e.m.f. in the output circuit which is proportional to the input current. The nature of this mechanism is not of importance to this discussion except that it is a one-way device. The mutual impedance M is the ratio of the e.m.f. generated in the output circuit to the current in the input circuit. This ratio may be adjusted by suitable means such as a potentiometer but is otherwise constant and includes no phase shift. The internal connections are assumed to be such that when the input terminal 1 is positive to 2 the e.m.f. in the output circuit tends to make terminal 3 positive with respect to 4.

Series Negative Resistance

In Fig. 2 the input and output circuits of the ideal amplifier are connected in series with each other to a source of e.m.f. E and a re-

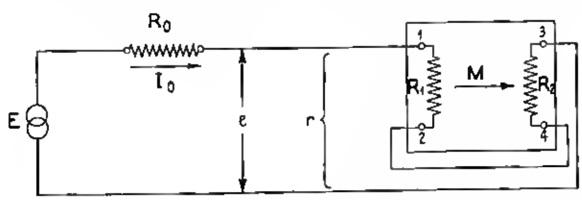

Fig. 2—One-way amplifier connected as a series negative resistance.

sistance R_0 in such fashion that the e.m.f. in the output circuit of the amplifier tends to increase the current. Assume now that the e.m.f. E is applied and a current I_0 flows in the series circuit.

$$E + (M - R_0 - R_1 - R_2)I_0 = 0. \tag{1}$$

The drop across the amplifier is:

$$e = (R_1 + R_2 - M)I_0, \tag{2}$$

and the net resistance of the whole amplifier is:

$$r = \frac{e}{I_0} = R_1 + R_2 - M. \tag{3}$$

It may aid in understanding the behavior of this system to assume, first, that M is zero so that the circuit consists simply of the three positive resistances R_0, R_1 and R_2 in series and then consider what happens as M is gradually increased. The e.m.f. appearing in the output circuit of the amplifier acts to reduce the drop e across the terminals 1, 3 and to increase the current I_0. The e.m.f. E must be reduced if the current is to be kept constant. The curves of Fig. 3 show how the resistances and current vary as M changes, E being constant.

When $M = R_1 + R_2$ the drop e and the resistance r become zero. The amplifier then ceases to take power from the circuit and supplies its own losses. If this condition could be exactly obtained the terminals 1, 3 might be short-circuited and the e.m.f. E removed, without changing the current which would continue to flow in the amplifier. If, however, the e.m.f. were removed or the circuit opened without short-circuiting the terminals of the amplifier the current in the input circuit, and, consequently, the e.m.f. in the output circuit of the amplifier would disappear and the system would become inactive.

If, now, M is further increased so that it approaches $R_0 + R_1 + R_2$ the current increases indefinitely, or the e.m.f. E required to sustain the current at a given value approaches zero. Under these conditions the drop e and the resistance r become negative and the amplifier supplies not only its own losses but also part of the energy dissipated by the resistance R_0. It does so under the control of the e.m.f. E, however, and if this e.m.f. is removed the system becomes inactive as before. At the limit when $M = R_0 + R_1 + R_2$, the amplifier supplies all the losses in the system and any current I_0, once started, continues indefinitely.

This ideal condition is not realized in practice. Either M is slightly too small, in which case the current decreases when E is removed, or it

is too large so that any value of E however small starts a current which thereafter increases because the amplifier supplies more than enough energy to sustain the current. This increase continues until checked by the inability of the amplifier to deal with larger currents. In effect M is reduced to the point where r is again equal to $-R_0$, after which the current continues at a constant value.

The arrangement shown in Fig. 2 can therefore be made to provide any negative resistance between $r = 0$ and $r = -R_0$ without causing instability or a tendency to sing. *Such a system is stable when the algebraic sum of all the resistances in series in the circuit is positive.* This behavior is typical of a large number of arrangements that are able to furnish negative resistances. All such arrangements will be referred to as *series negative resistances* to distinguish them from another type which will be described below.

It should be noted that if the sign of M is reversed, for example, by interchanging the two wires connected with the output terminals 3, 4, no negative resistance results. As M increases, the current I_0 decreases, or the e.m.f. E must be increased to maintain the current, but no matter how large M is made, the direction of the drop e and sign of the resistance r do not change though the latter approaches ∞.

The Unstable Condition

So far nothing has been said as to the nature of the e.m.f. E. In the ideal case, when the system is stable, the current wave is a copy of the voltage wave as in any circuit having a pure resistance. What happens when the circuit is unstable depends upon the nature of the amplifier or other device used to produce the negative resistance and not upon the e.m.f. E. This may be of any kind and of minute size, such as that resulting from thermal agitation in the resistances forming part of the apparatus. If the amplifier is able to amplify direct currents, the resulting disturbance may be a direct current limited only by the ability of the apparatus to supply energy to the circuit. Where transformers, condensers, etc., are involved the disturbance settles down to an alternating current which may contain many harmonics or may be almost a pure sine wave. These effects are called "singing." The final frequency, amplitude and wave shape depend upon the makeup of the apparatus in a way which is beyond the scope of this paper.

Shunt Negative Resistance

By connecting the terminals of the ideal one-way amplifier in parallel as shown in Fig. 4, a negative resistance will be obtained which is typical of the second type or *shunt negative resistance.*

Referring to Fig. 4, the current in the input circuit of the amplifier is:

$$I_1 = \frac{e}{R_1}. \tag{4}$$

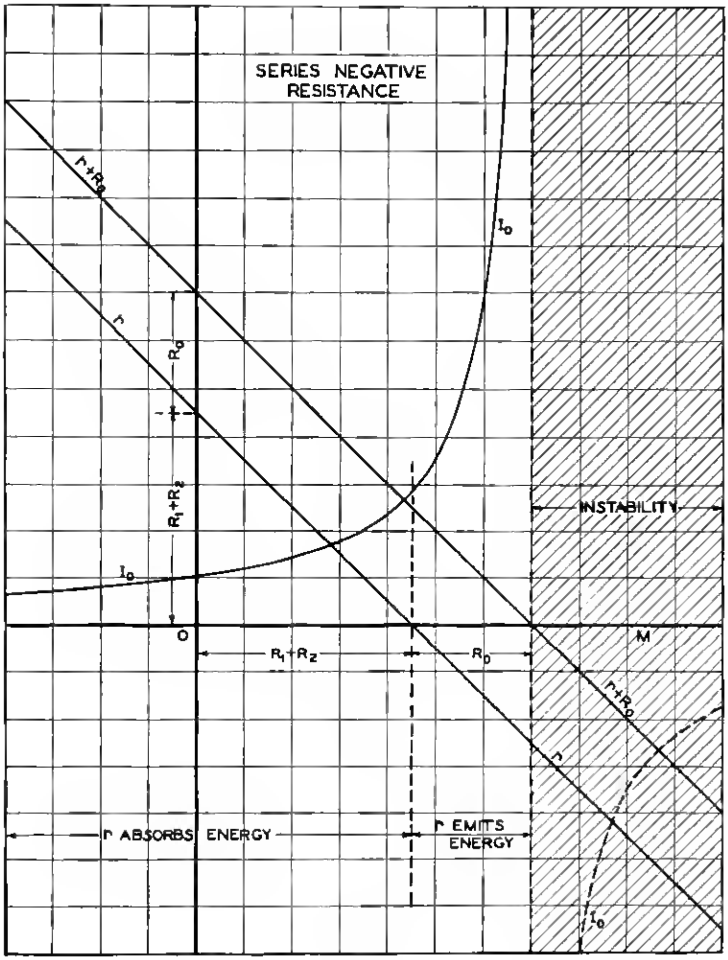

Fig. 3—Curves illustrating properties of series negative resistances.

The current in the output circuit is:

$$I_2 = \frac{e - MI_1}{R_2} = \frac{R_1 - M}{R_1 R_2} e, \tag{5}$$

and the current in the main circuit is:

$$I_0 = \frac{E - e}{R_0} = I_1 + I_2 = \frac{R_1 + R_2 - M}{R_1 R_2} e, \tag{6}$$

from which

$$r = \frac{e}{I_0} = \frac{R_1 R_2}{R_1 + R_2 - M}, \tag{7}$$

and the applied voltage E is:

$$E = I_0(R_0 + r) = \left[1 + \frac{R_0(R_1 + R_2 - M)}{R_1 R_2} \right] e. \tag{8}$$

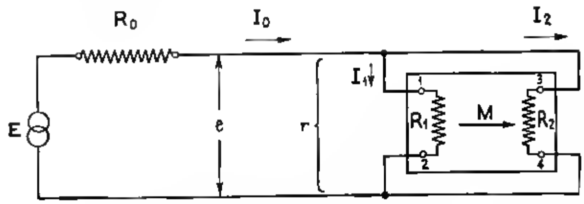

Fig. 4—One-way amplifier connected as a shunt negative resistance.

With this arrangement, the e.m.f. generated in the output circuit of the amplifier opposes the current I_2 due to the e.m.f. E, and as M increases, the current I_0 in the main circuit decreases and the resistance of the amplifier increases. The curves of Fig. 5 show how the resistances and current vary as M changes, E being constant. To keep I_0 constant, it would now be necessary to increase E.

When $M = R_1$ the current I_2 becomes zero.

When $M = R_1 + R_2$ the current I_0 falls to zero, the potential $e = E$, the current I_2 has reversed in direction, the resistance $r = \infty$ and the amplifier just supplies its own losses. If the circuit outside the amplifier is now opened, the condition of the amplifier is the same as when the short circuit was applied to Fig. 2 and the current circulating in the amplifier will continue. If E is removed without opening the circuit, R_0 will draw energy from the amplifier, thus reducing I_1 and causing all currents and voltages to disappear. The amplifier is still under the control of the e.m.f. E.

For the arrangement of Fig. 4 to become unstable it is necessary for the amplifier to maintain or increase the voltage e after the controlling e.m.f. E is removed. For the amplifier to maintain the voltage e it is necessary that:

$$e = \frac{e}{R_1} M \frac{\dfrac{R_0 R_1}{R_0 + R_1}}{\dfrac{R_0 R_1}{R_0 + R_1} + R_2}, \tag{9}$$

from which

$$M = R_1 + R_2 + \frac{R_1 R_2}{R_0}, \tag{10}$$

and from (7),

$$r = -R_0. \tag{11}$$

Hence if

$$R_1 + R_2 < M < R_1 + R_2 + \frac{R_1 R_2}{R_0}, \tag{12}$$

the impedance r is a negative resistance greater in magnitude than R_0 but the system cannot sing because the amplifier cannot maintain or increase

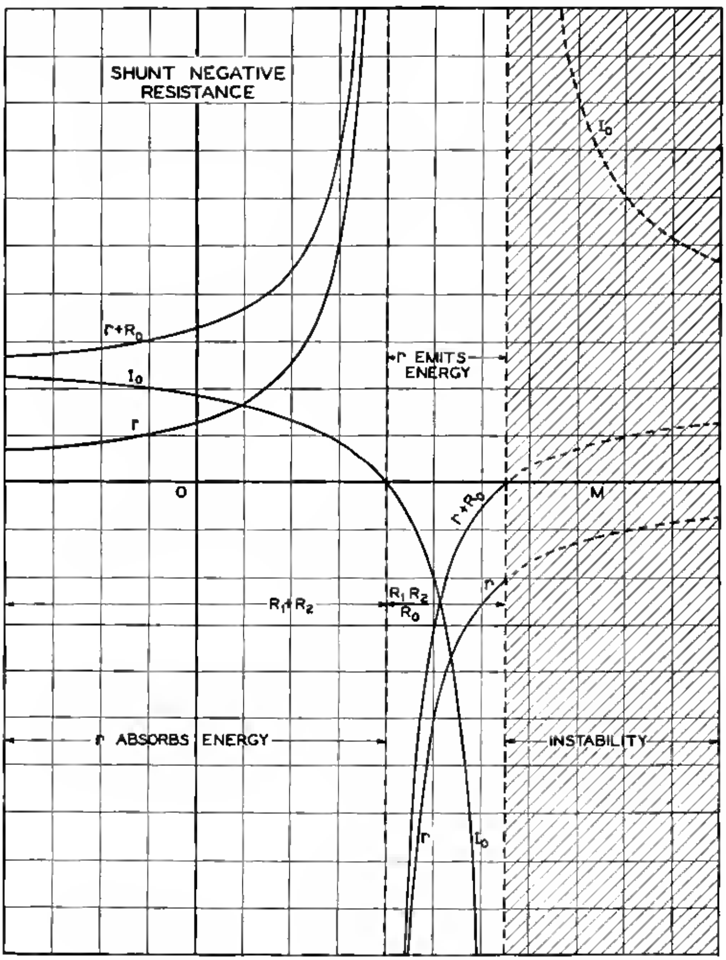

Fig. 5—Curves illustrating properties of shunt negative resistances.

the voltage e after E is removed, even though the current I_0 flows against E and the source is receiving energy from the amplifier.

If M becomes greater than the upper limit given by equation (10) the system passes out of control by the e.m.f. E and becomes unstable or sings. By short-circuiting the terminals 1, 2, it would be possible to increase M until it is greater than the value given by equation (10) which would make r numerically smaller than R_0. On removing the short circuit, however, a disturbance would begin and grow until checked by the limitations of the amplifier so that, in effect, M would be reduced and r again made equal to $-R_0$.

If M is reversed in sign, for example, by interchanging the two wires connected to the output terminals 3, 4, no negative resistance results. As M increases, the current I_0 increases. The resistance r decreases, approaching zero as M becomes indefinitely great.

From these facts it is seen that a negative resistance of any desired value may be inserted in a circuit having any positive resistance R_0 provided that the inserted resistance has the characteristics of the series type when the inserted negative resistance is numerically smaller than the positive resistance or the characteristics of the shunt type when the negative resistance is numerically larger than the positive resistance.

Other Forms of Negative Resistance

All known devices for producing negative resistance fall into one or the other of the two classes described above.

Arrangements are known which exhibit one type of negative resistance at one pair of terminals and the other type at a different pair but not both types at the same pair of terminals at the same time.

Certain apparatus involving gaseous conduction or electronic discharge exhibit negative resistance effects. Fig. 6, for example, shows an arc burning between two electrodes which are connected in series with a resistance and inductance serving as ballast to a source of d-c. power. The ballast serves to stabilize the arc and hold the current drawn from the source constant and also to prevent the passage of alternating current through the source from the arc. The arc has a positive resistance with respect to the d-c. circuit, since it consumes d-c. power, but this resistance varies with the current in such a way that an increase of current is accompanied by a reduction of the potential drop across the arc.

If an alternating current is superimposed upon the direct current through the arc by means of the taps a and b it encounters a negative resistance. If a circuit consisting of a resistance R, inductance L and

condenser C is bridged across the arc as shown and the resistance is made large, nothing occurs, but if the resistance is reduced to a certain critical value a state of oscillation is established. This oscillation causes an alternating current to flow through the resonant circuit and the arc. If the oscillation is of audible frequency the arc will emit a

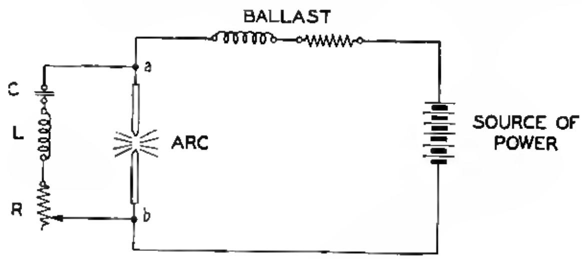

Fig. 6—Arc as a negative resistance.

singing or whistling sound. This property of the arc has found useful application as a generator of high-frequency oscillations in the Poulsen arc used in radiotelegraphy. The negative resistance of the arc has series characteristics as oscillations will not occur if there is an excess of positive resistance in the oscillating circuit.

The dynatron,[1] on the other hand, has shunt characteristics as it is unstable when the external resistance is made large.

Negative Resistances of the Ideal 21-Type Circuit

Fig. 7 shows the ideal one-way amplifier of Fig. 1 connected with an ideal hybrid coil to form a 21-type repeater circuit. The ideal hybrid

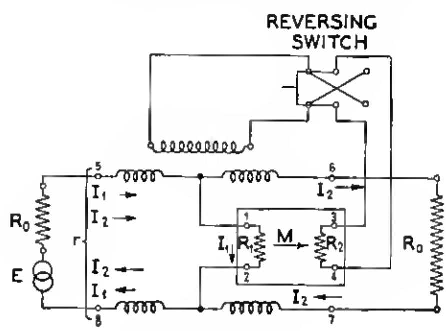

Fig. 7—Ideal 21-type circuit.

coil is assumed to have windings of zero resistance, no leakage reactance, no capacitance in or between the windings, no core loss and negligible exciting current.

[1] See "The Dynatron," by A. W. Hull, *Proc. I. R. E.*, February, 1918.

This 21-type circuit is connected between two equal resistances, R_0. $R_1 = \frac{1}{2}R_0$ and $R_2 = R_0$, assuming that the hybrid coil is designed for equal impedances at the two pairs of line terminals and the drop terminals. If an e.m.f. E acts in series with the resistance R_0 at the left side of the repeater and the mutual impedance M of the amplifier is zero, a current,

$$I_1 = \frac{E}{2R_0}, \tag{13}$$

flows at the left hand terminals 5, 8 of the hybrid coil and in the input circuit of the amplifier. One-half of the power entering the repeater is absorbed in the input resistance R_1 of the amplifier and the other half is absorbed in the output impedance R_2. In accordance with a well known property of the hybrid coil, no current will flow in the right-hand resistance R_0. At a given instant this input current may be assumed to have the direction indicated by the short arrows I_1. By increasing M the amplifier can be made active, causing an amplified current I_2 to flow in series through the line windings of the hybrid coil and the connected resistances R_0. By throwing the reversing switch to one side I_2 may be made to flow in the same direction as I_1 at the terminals 5, 8 as indicated by the long arrows marked I_2. For convenience, this will be referred to as the "direct connection." Changing the reversing switch changes the direction of I_2 with respect to I_1, giving the "reverse connection." As the hybrid coil is balanced, the output power of the amplifier does not react upon the input circuit. Putting A for the amplifying ratio of the 21-type circuit,

$$A = \frac{I_2}{I_1}. \tag{14}$$

The total current flowing at the terminals 5, 8 is:

$$I_0 = I_1 + I_2 = \frac{E}{2R_0}(1 + A), \tag{15}$$

and the active resistance of the 21-type circuit is:

$$r = \frac{E}{I_0} - R_0 = \frac{1 - A}{1 + A} R_0. \tag{16}$$

As the amplification is increased, the current I_0 increases while r falls to zero and becomes negative, thus exhibiting series characteristics. If A is increased without limit, r approaches $-R_0$ in magnitude but

cannot reach it, while A remains finite. That is, the system shown in Fig. 7 cannot sing. This is also obvious from the fact that the hybrid coil is balanced. However, the resistance r does not depend upon holding R_0 at the terminals 5, 8 constant. If the resistance at the terminals 5, 8 is reduced to a lower value R_0', while that at the terminals 6, 7 is held constant at R_0, the output energy of the amplifier is permitted to reach the input terminals 1, 2 and when $-R_0' = r$ instability or singing can occur.

Throwing the reversing switch to give the reversed connection has the effect of reversing the sign of the amplification A. The total current I_0 at the terminals 5, 8 decreases to zero, reverses and increases as A increases, while r increases, passes discontinuously from $+\infty$ to $-\infty$ and decreases in magnitude. Again r approaches $-R_0$ as A increases indefinitely, but cannot reach it. However, by increasing the resistance connected to the terminals 5, 8 to a higher value R_0' such that $-R_0' = r$, instability will occur. The reversed connection thus gives a negative resistance of shunt characteristics.

Referring to Fig. 7 and assuming that the switch is thrown to give the directions of current flow indicated by the arrows, transfer the e.m.f. E to the right-hand end of the diagram. This change will not change the direction of I_1 in the input circuit of the amplifier or the direction I_2 at any point. The current I_1 will now be found at terminals 6, 7 instead of 5, 8 and will be flowing in the direction opposite to I_2. From this it will be seen that a 21-type circuit which is direct-connected with reference to terminals 5, 8, giving a series type negative resistance, will be reverse-connected, and give a shunt type negative resistance at the opposite terminals 6, 7. Changing the reversing switch reverses the conditions at both pairs of terminals.

Non-Ideal Devices

The discussion has so far been confined principally to certain ideal conditions which can only be approximated in practice, but consideration of these simple cases will serve to illustrate the important fundamental properties of negative resistances and the requirements that must be met to insure stable operation.

To obtain a pure negative resistance from a one-way amplifier or from a 21-type repeater circuit requires that there shall be no phase shift in the process of amplification. This can only be approximated in practice because even a resistance coupled amplifier system involves small inductances and capacitances in the tubes and wiring which produce phase shifts at high frequencies. Commercially practicable trans-

formers, choke coils, and condensers which are so useful in assemblages of apparatus which include vacuum tubes further limit the range of frequency over which an approximately pure negative resistance may be obtained. In some cases, this may not be a serious disadvantage. Suppose, for example, it is desired to reduce the effective resistance of a series resonant circuit in order to obtain more nearly ideal performance at the resonant frequency. It would be sufficient to arrange a negative resistance in series with the resonant circuit which would produce the desired result at and near the resonant frequency and which would produce no harmful effect at other frequencies even though it departed widely from the value at the resonant frequency. In other cases the variation of the negative resistance with frequency and the introduction of reactive components do no serious harm and may even be quite useful as in the case of the twin 21-type repeater to be described below. In still other cases the difficulties of producing a negative resistance of satisfactory characteristics may be very great.

General Negative Impedance

The arrangements described above produce under ideal conditions pure negative resistances.

It has been shown by R. C. Mathes and H. W. Dudley that it is possible to produce any desired negative impedance provided that the positive of this impedance can be constructed in the form of a network.

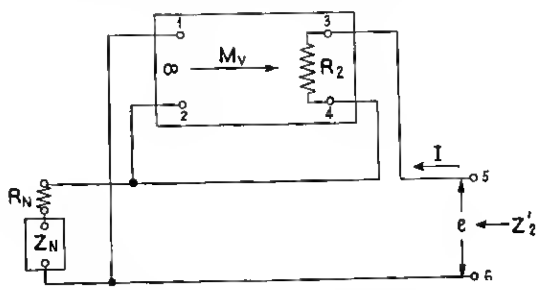

Fig. 8—Series type negative impedance.

Fig. 8 shows in simplified form the arrangement invented by Mathes, and Fig. 9 shows the arrangement due to Dudley. Each of these arrangements requires a distortionless one-way amplifier whose input impedance (terminals 1, 2) is substantially infinite. This condition is easily approximated by using vacuum tubes. In discussing the behavior of such arrangements, it is necessary to use the ratio, M_v, of

the e.m.f. generated in the output circuit of the amplifier to the voltage impressed on its input terminals, instead of the mutual impedance of the amplifier, because the input current is negligibly small. This ratio may be adjusted by some suitable means such as a potentiometer.

Referring to Fig. 8, let Z be the positive of any desired negative impedance such that a network having the impedance, $Z_N = Z/M_v - 1$, may be constructed of physically available parts, M_v being a real

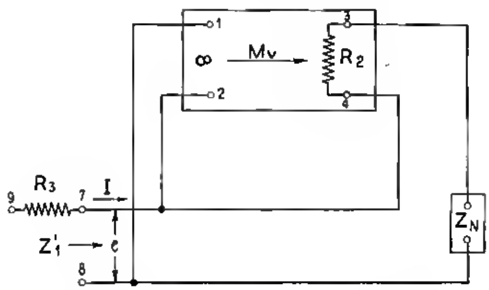

Fig. 9—Shunt type negative impedance.

number greater than 1. $R_N = R_2/M_v - 1$ is a pure positive resistance. Next assume that a current, I, is flowing through the circuit between terminals 5 and 6. The e.m.f. generated in the output circuit of the amplifier is $(R_N + Z_N)IM_v$. It acts in the direction which tends to increase the current. The voltage e required at the terminals 5, 6 to produce this current is, then,

$$e = (R_N + Z_N + R_2)I - (R_N + Z_N)IM_v, \tag{17}$$

from which the impedance Z_2' is:

$$Z_2' = \frac{e}{I} = -Z, \tag{18}$$

which is the desired negative impedance. Due to the arrangement of the circuit this impedance has series characteristics.

Referring to Fig. 9, Z_N is a positive network. Assuming that an e.m.f. e is applied to the terminals 7, 8, the e.m.f. generated in the output circuit of the amplifier is eM_v which acts in opposition to e to reduce or reverse the current. The current at the terminals 7, 8 is, then,

$$I = \frac{e - eM_v}{R_2 + Z_N}, \tag{19}$$

and the impedance Z_1' at the terminals 7, 8 is:

$$Z_1' = \frac{e}{I} = \frac{R_2}{1 - M_v} - Z, \tag{20}$$

which consists of the desired negative impedance $-Z$ and a negative resistance if $M_v > 1$. By connecting the positive resistance, $R_3 = R_2/M_v - 1$, in series with Z_1' this negative resistance is neutralized and the desired negative impedance is found between the terminals 8 and 9. This impedance has shunt characteristics.

In both of these arrangements it is possible, without changing the constants of the network Z_N, to give the negative impedance any desired magnitude by adjusting the value of M_v and making the corresponding change in the resistance R_N or R_3.

Boosters

The name "booster" has been applied to a negative impedance of suitable characteristics connected in series with or bridged across a telephone circuit in order to introduce energy when a wave passes and so produce a transmission gain. Such devices have certain interesting theoretical properties.

Series Booster

Fig. 10 shows an impedance Z_s connected in series between the two parts of a telephone line having the characteristic impedance Z_0.

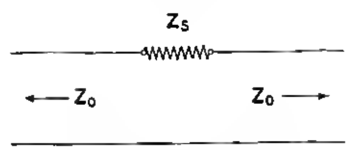

Fig. 10—The series booster.

Assume first that Z_s is a positive impedance having the same angle as Z_0 and that a wave is traveling over the line, for example, from left to right. The effect of the inserted impedance is to reduce the current in the line wires at the point of insertion, weakening the wave that passes on to the receiver and causing a reflected wave to return to the source. The transmission loss [2] caused by the inserted impedance is:

$$L = 20 \log_{10}\left(1 + \frac{Z_s}{2Z_0}\right), \tag{21}$$

[2] The values of losses, return losses and gains will be expressed in decibels (db) throughout this paper.

and the return loss [3] due to the irregularity is:

$$S = 20 \log_{10}\left(1 + \frac{2Z_0}{Z_s}\right). \tag{22}$$

If, now, Z_s is made a negative impedance of the series type smaller in magnitude than $2Z_0$, the potential difference between its terminals reverses in sign, the current at the point of insertion increases, the loss becomes a gain and the reflected wave reverses in sign. As Z_s approaches $-2Z_0$, the transmitted and reflected waves increase until singing occurs; but the reflected wave is always smaller than the transmitted though they approach each other as the gain increases. Such a booster, therefore, causes a smaller returned wave or echo than an ideal 21-type repeater circuit working between ideal line impedances which always returns a wave toward the source which is equal to that transmitted toward the receiver.

The series booster would also operate if Z_s were made a shunt type negative impedance greater in magnitude than $2Z_0$, but in this case the current at the booster and the wave traveling toward the receiver would be reversed in phase and the reflected wave or echo would be greater than the wave traveling toward the receiver. This arrangement would, therefore, give greater echoes for a given gain than a 21-type repeater. The curves of Fig. 12 show the relation between the return loss and transmission gain for these boosters in comparison with a 21-type repeater.

The echoes referred to above are, of course, those inherent in the operation of the devices described and would not occur if a 22-type repeater were used with perfect lines. Echoes due to line irregularities would be amplified to the same extent by boosters as by any other type of two-way repeater giving the same gain.

Shunt Booster

Fig. 11 shows an impedance Z_b bridged across the line. The effect of this impedance is to reduce the wave traveling toward the receiver, causing a transmission loss,

$$L = 20 \log_{10}\left(1 + \frac{Z_0}{2Z_b}\right), \tag{23}$$

and causing a reflected wave to return to the source with a return loss,

$$S = 20 \log_{10}\left(1 + \frac{2Z_b}{Z_0}\right). \tag{24}$$

[3] When a wave is partially reflected at an irregularity the relation between the reflected part and the original wave, expressed in decibels, is called the return loss.

In this case the current in the line leading toward the source is increased; that is, the reflected wave is of opposite phase to that reflected by an impedance in series with the line.

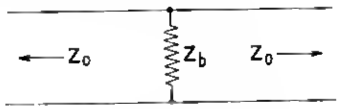

Fig. 11—The shunt booster.

If Z_b is made a negative impedance with shunt characteristics and greater in magnitude than $Z_0/2$, the current through Z_b reverses in sign, the wave transmitted toward the receiver increases, the transmis-

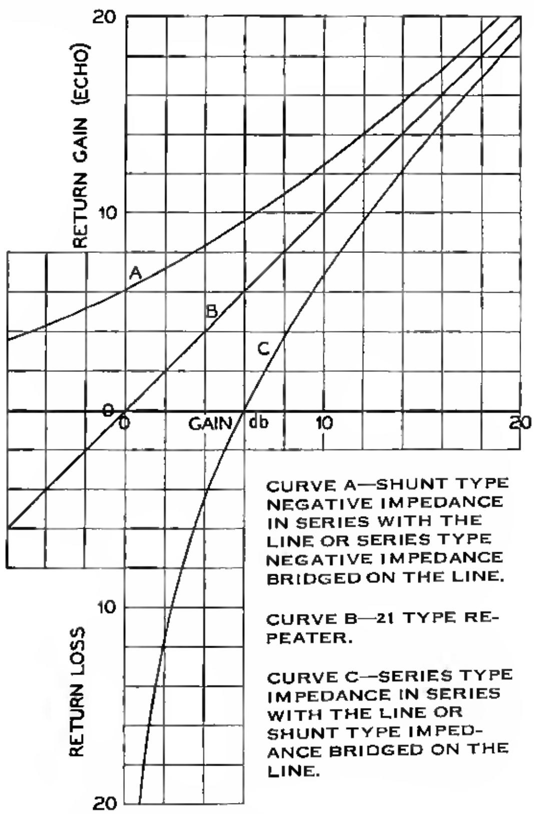

Fig. 12—Echoes caused by boosters and 21-type repeaters.

sion loss becoming a gain and the reflected wave reverses in sign, thus reducing or reversing the current in the line leading toward the source. The relation between the magnitude of the echo and the gain is the

same as for the series type booster described above except that the reflected waves are opposite in phase. This makes it possible to eliminate the echo by combining two boosters in one repeating device as described below.

The shunt booster would also operate if Z_b were made a series type negative impedance smaller in magnitude than $Z_0/2$, but in this case the wave traveling toward the receiver would be reversed in phase and the echo wave would be greater than the transmitted wave.

Singing Points of Various Forms of Repeaters

When a line of characteristic impedance Z_0 has a certain return loss S_l, its impedance will lie between a maximum value of mZ_0 and a minimum of Z_0/m where

$$S_l = 20 \log_{10} \frac{m+1}{m-1}. \tag{25}$$

If two pieces of such a line are joined through a repeating device the high and low impedances may combine in three different ways which give the greatest tendency to sing with different types of apparatus.

The series type negative impedance, whether connected in series with or across the line, has the greatest tendency to sing when the minimum impedances of both lines occur at the same frequency and the shunt type negative impedance has the greatest tendency to sing when the maximum impedances occur at the same frequency. The 21-type repeater has the greatest tendency to sing when the maximum impedance of one line and the minimum of the other occur at the same frequency, the internal connections of the repeater determining which impedance must be high. In the 22-type repeater any of these combinations may be the worst, depending upon the internal arrangement of the repeater circuit.

The series booster (with series type negative impedance) will sing when

$$Z_s + \frac{2Z_0}{m} = 0. \tag{26}$$

Substituting Z_s obtained from this relation in equation (21) and remembering that the loss L becomes a gain G_s when Z_s is negative, the gain which will produce singing is:

$$G_s = 20 \log_{10}\left(1 - \frac{1}{m}\right). \tag{27}$$

This gain is, of course, the gain which a booster having the impedance Z_s obtained from equation (26) would produce when connected between two impedances Z_0. The actual gain of the booster, like that of any other type of repeater approaches infinity as the singing condition is approached.

The shunt booster (with shunt type negative impedance) will sing when

$$Z_b + \frac{mZ_0}{2} = 0. \tag{28}$$

Substituting the value of Z_b from this equation in equation (23) shows that the relation given in equation (27) also holds for the shunt type booster.

It is well known that when a 22-type repeater giving the gain G_{22} in each direction is connected between two lines having the return loss S_l singing will occur when

$$G_{22} = S_l, \tag{29}$$

if the worst combination of unbalances occurs.

It is also well known that under similar conditions the gain of a 21-type repeater is:

$$G_{21} = S_l - 6\text{db}, \tag{30}$$

because of the fact that waves reflected from the irregularities in both lines combine in the input circuit of the amplifier.

The curves of Fig. 13 show the singing gain as a function of line return loss for boosters, 21-type and 22-type repeaters. These curves together with the curves of Fig. 10 indicate that ideal boosters consisting of series type negative impedances in series with the line or shunt type negative impedances bridged across the line have properties intermediate between those of 21 and 22-type repeaters with respect to the amount of echo and margin against singing for a given transmission gain. These properties are particularly favorable at low gains.

In practice, however, it is usually necessary to limit the amplification to a definite band of frequencies in order to avoid the effect of impedance unbalances and interfering disturbances at frequencies outside these limits. This must be accomplished by the use of inductance and capacitance in the form of filters, transformers, choke coils or condensers. It is also desirable to couple the series booster to the line by means of a transformer having two equal windings, one in each line conductor, to enable one booster mechanism to operate without unbalancing the line and to permit the passage of low frequency signaling waves from one part of the line to the other without interference

from the booster. For similar reasons, condensers must be connected in series with the shunt booster when it is bridged on the line. These devices, particularly the filters, shift the phase of the amplified waves, and modify the negative impedances so that the gain varies with frequency in the useful range to a greater extent than is the case with the 21 and 22-type repeaters and the echoes are increased. This variation of gain is due to the fact that the booster, in effect, superimposes an amplified wave upon the wave that would exist if the

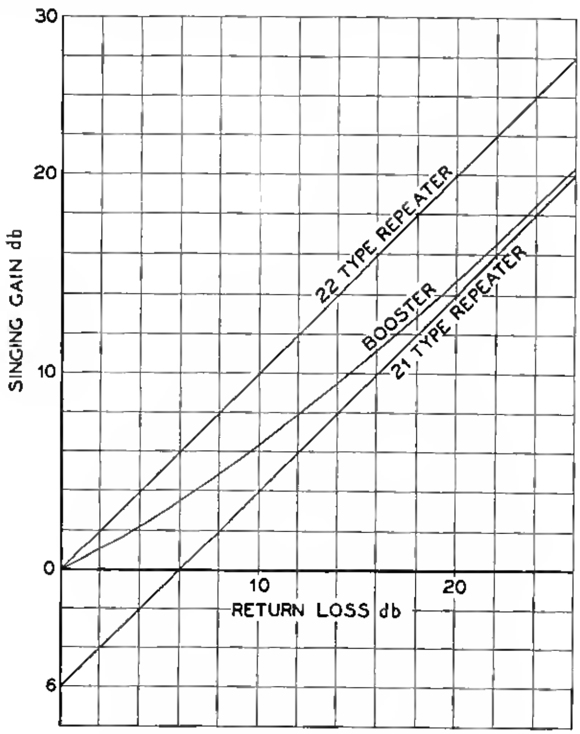

Fig. 13—Singing gains of boosters and repeaters.

booster were removed. The received wave, being the resultant of these two waves, varies with the phase angle between them.

It should also be noted that boosters do not avoid the problem of matching line impedances or the difficulties due to impedance irregularities in the line. To obtain a gain that is constant over a wide range of frequencies, the negative impedance must be fitted to the line impedance over this range and there must be no large irregularities. It will be shown below that most of the difficulties described above may be avoided by using a series and a shunt booster in combination.

Negative Impedances Arranged in T or Π Networks

It has been pointed out by G. A. Campbell, H. Mouradian,[4] and possibly by others, that three negative impedances can be grouped into a T or a π network which may be inserted in a telephone line. Such a network is able to amplify waves traversing the line without causing echoes if the values of the impedances are suitably chosen. In order to avoid singing, the impedances in series with the line must be of the series type, and those bridged across the line, of the shunt type.

A Double Booster

Fig. 14 shows a network of impedances connected between two pieces of telephone line having the characteristic impedance Z_0. These lines are assumed at first to be free from irregularities. The branches ac and bc are fixed networks, each having the impedance Z_0. Branches ab and cd are networks whose impedances can be varied reciprocally from the value Z_0, that is, if one impedance is multiplied by a factor

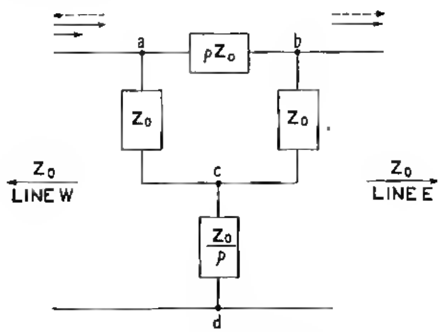

Fig. 14—Double booster.

ρ, the other is divided by the same factor. The factor ρ may be positive or negative, and may be complex. Branches ab, ac, cd and the line E may be considered as forming the arms of a Wheatstone bridge, of which the branch bc is one diagonal and the line W is the other. This bridge is balanced; consequently, the impedance connected to the line W consists of two parallel circuits, one comprising the branch ab in series with the line E and the other comprising the branches ac and cd in series. This impedance is independent of ρ, being equal to Z_0. By symmetry, the impedance connected to the line E is also equal to Z_0, so no reflection occurs at the terminals of the network.

Assuming that a wave arrives, for example, over the line W and is

[4] "Long Distance Transmission Problems," by H. Mouradian, *Journal of the Franklin Institute*, Vol. 207, No. 2, February, 1929.

transmitted to the line E, the ratio of the voltage across the terminals a, d to that impressed on the line E is $(1 + \rho)/1$ and the transmission loss through the network is:

$$L = 20 \log_{10} (1 + \rho). \tag{31}$$

This loss becomes a gain when ρ becomes negative and the network acts as an amplifier.

Examination of Fig. 14 shows that the branches ab and cd are each connected to a constant impedance Z_0, hence, if ρ lies between 0 and -1, the branch ab must be a series type negative impedance and cd of the shunt type. If ρ lies between -1 and $-\infty$, these types must be interchanged.

The physical behavior of this network may be readily understood if the properties of negative impedances are kept in mind. At first let ρ be infinite and assume that a wave arrives over the line W. This wave tends to produce a current in the upper conductor which at a given instant flows in the direction indicated by the short solid arrow. This wave will be absorbed by the impedance Z_0 of the branch ac. If now ρ is made negative the series type negative impedance in the branch ab will cause additional currents to flow in the lines E and W the directions and relative magnitudes of which are indicated by the longer solid arrows. The shunt type negative impedance in the branch cd tends to produce currents having the directions indicated by the dotted arrows, thus further increasing the wave in the line E but annulling the effect of the series type impedance in the line W. The network of Fig. 12, therefore, amplifies waves traveling in either direction without causing echoes to return to the source. It resembles, in this respect, a 22-type repeater, but it cannot give different gains in the two directions.

Putting a shunt type impedance in the branch ab and a series type in the branch cd would reverse the sign of the amplified wave.

For such a network to function as described above, it is not necessary for the ratio ρ to be independent of frequency. Phase shifts in the negative impedances are permissible provided they are kept equal so that the echoes will be eliminated. It is, therefore, possible to use filters and other apparatus to cause the gain to vary with frequency in a desired manner without encountering the troubles which occur in the single booster.

It is further possible to couple the series branch ab of the network of Fig. 12 to the line by means of a transformer and the bridged branch cd by means of a condenser without seriously altering the reciprocal

relation of these impedances. This provides a method for permitting low frequency signals to pass over the line without serious interference from the rest of the network.

The Twin 21-Type Repeater Circuit

A simplified diagram of a twin 21-type circuit is shown in Fig. 15. This consists of a line hybrid coil whose line windings are connected in series with the line conductors and two 21-type circuits. One pair of terminals of one of the 21-type circuits is connected with the drop winding of the hybrid coil which couples it effectively in series between the two parts of the line. A network N_S is connected to the remaining terminals which balances the impedance of the two parts of the line as seen from the 21-type circuit. One pair of terminals of the second 21-type circuit is connected to the bridge terminals of the hybrid coil which bridges it across the line. A network N_B is connected to the remaining terminals which balances the impedance of the two parts of the line as seen from the bridge. The internal connection of the series 21-type circuit is direct with respect to the line hybrid coil and the bridged circuit is reversed.

At first, assume that the potentiometers of the two 21-type circuits are turned down and that a wave arrives at the W line terminals of the twin 21-type circuit. At the peak of the positive half-cycle, currents will flow in the line hybrid coil in the directions indicated by the arrows marked I. The passive impedances are chosen to fit the normal impedances at the drop and bridge terminals of the line hybrid coil; hence, none of this wave will reach the E end of the line.

Next, turn the potentiometer of the series 21-type circuit up until this circuit gives a gain. Due to the internal arrangement of this circuit, an amplified current will flow in the line conductors in the directions indicated by the large arrows marked I_{OS}. Little or none of this current will reach the bridged circuit because of the balance between the two parts of the line.

Finally, turn the potentiometer of the bridged 21-type circuit up until this circuit gives the same gain as the series circuit. Due to the internal arrangement of the bridged circuit, amplified currents will flow in the line conductors in the directions indicated by the arrows marked I_{OB}. These currents are equal in magnitude to those caused by the series 21-type circuit. In the line W the output currents annul each other so that echoes returning toward the speaker are suppressed while the currents in the line E co-operate and an amplified wave travels over the line E to the listener.

If the incoming wave arrives over the line E, the action is the same except that the direction of the current I_{OS} due to the series 21-type circuit is reversed with respect to the current I_{OB} which causes the amplified wave to travel toward the W end of the line.

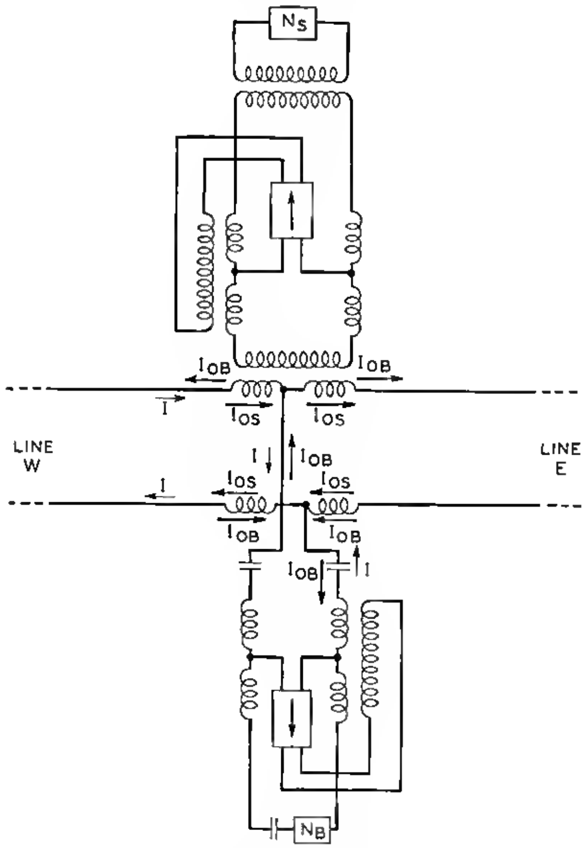

Fig. 15—The ideal twin 21-type repeater.

If the internal connection of both 21-type circuits is changed the direction of both output currents will be reversed. The action of the twin 21-type circuit will not be altered except that the phase of the amplified wave will be reversed. Changing only one of the 21-type circuits, however, will cause the amplified wave to travel toward the speaker as a powerful echo and will prevent transmission toward the listener.

Effect of Phase Shift

In the foregoing description the discussion has been simplified by assuming ideal transformers and 21-type circuits. It is sufficient,

however, that at any frequency where amplification occurs, the gains and phase shifts of the two 21-type circuits should be equal. This insures that the echoes will balance out and that the maximum output power will be directed toward the listener. To accomplish this it is merely necessary to make the corresponding parts of the two circuits alike within the allowable tolerance. Filters, condensers and other devices may be used as required provided that the corresponding parts in the two 21-type circuits are nearly enough alike.

Referring to Fig. 15, the series 21-type circuit is coupled to the line inductively by the line hybrid coil and the shunt circuit is connected through condensers. This arrangement makes it possible for the line conductors to be joined through the windings of the twin 21-type circuit without a conductive bridge across the line, and so provides the desired path for d-c. impulses or low frequency alternating current.

The inductance of the line hybrid coil cannot, of course, be infinite. Practically, it must be a compromise between the opposing requirements that it shall be low enough not to interfere seriously with the transmission of low frequency signaling impulses or transfer too much of their energy to the series 21-type circuit and that it shall be high enough to prevent too great a transmission loss at the lower frequencies of the voice range. Due to this finite inductance, the amplified voice currents from the series 21-type circuit will be shifted in phase at the lower frequencies.

The capacitance of the condensers in series with the bridged 21-type circuit is similarly limited, and shifts the phase of the amplified voice currents from that circuit. These two shifts are in the same direction which makes it possible to keep the amplified currents from the two 21-type circuits in phase and prevent the production of echoes.

The transmission loss and phase shift due to the finite inductance of the line hybrid coil will be approximately equal to the loss and phase shift due to the condensers when

$$\frac{L_1}{C_1} = \frac{L}{C} = R^2, \tag{32}$$

in which

L_1 = Inductance of the whole line winding of the line hybrid coil, with the drop open.

C_1 = Capacitance in series with the bridged circuit.

L = Inductance per unit length of the line.

C = Capacitance per unit length of the line.

R = Nominal impedance of the line.

When two condensers are used in series, as shown, to keep the circuit balanced, each one must, of course, have a capacitance $2C_1$.

It would be possible to carry this principle still further, if necessary, so that anything introduced between the series 21-type circuit and the line which results in adding impedance in series or shunt with the series 21-type circuit can be matched by adding suitable impedance in shunt or series, respectively, with the bridged 21-type circuit. Similarly, anything which affects the impedance of the bridged circuit can be matched by a corresponding addition to the series circuit. In order for these additional impedances to match, the following relation must be established at all frequencies in the useful range:

$$z_S \times z_B = R^2, \tag{33}$$

in which z_S is an impedance effectively in series, or parallel, with the series 21-type circuit and z_B an impedance effectively in parallel, or series, respectively, with the bridged 21-type circuit. The value z_S is referred to the line windings of the line hybrid coil, that is, if the element contributing this impedance is connected to the drop winding of the line hybrid coil its actual impedance must be multiplied by the square of the turn ratio of the entire line winding to the drop winding to obtain z_S.

If more than one part of the series 21-type circuit must be compensated by corresponding parts of the shunt circuit, it is necessary that the corresponding parts be arranged in the same order between the line and the 21-type circuits.

Special Properties of the Twin 21-Type Circuit

The twin 21-type repeater differs in a number of important respects from the 22-type repeater and others that have been used or proposed in the past. It is essentially a network of impedances two of which include negative resistance components. These are the two 21-type circuits. Each 21-type circuit is connected to the line by only one pair of terminals through which the input wave enters and the amplified wave leaves it; hence, it may be treated as a single impedance which has a negative resistance component. It follows from this that the twin 21-type circuit follows the *reciprocal law*, and that the gain at any frequency is the same for both directions of transmission. This is true even if the two 21-type circuits are not set for the same gain. If the gains of the two circuits are different the amplified current wave will be the sum of the current waves from the two 21-type circuits (as measured in milliamperes or other current units) and an echo equal to the difference will travel toward the speaker.

Another difference lies in the fact that both amplifiers work at the same time. Even though the echo waves are cancelled out their energy is not lost, but is added to the amplified wave. The output current is twice, and the output power is four times what either 21-type circuit acting alone would send toward the listener. In the 21 or 22-type circuit, only half the output power of one amplifier reaches the line, the other half being absorbed in the opposite line or in the network. The output power is, therefore, 3 db less than that which the amplifier actually produces. In the twin 21-type circuit one-half the output power of each amplifier is also absorbed in a network, but the remaining halves are combined in the output wave. Consequently, the total output is equal to that of one amplifier. For this reason, with a given size of vacuum tube, the twin 21-type circuit can deliver twice as much useful power, or 3 db larger volume to the line, than either the 21 or 22-type repeater.

Push-Pull Effect

If the connection between the line hybrid coil and either of the 21-type circuits is transposed, the directions of current flow in the 21-type circuit are all reversed, but the directions of the input and output currents in the line conductors are not affected. If the amplifiers are perfect, such a transposition will have no effect upon the operation of the twin 21-type circuit. When vacuum tubes are used as amplifiers, however, there is a certain amount of distortion due to the curvature of the operating characteristics of the tubes.

If the connections are so arranged that the grids of the tubes in both of the 21-type circuits receive positive potentials from the incoming wave during the same half-cycle, this distortion will appear in the output wave of the twin 21-type circuit. If, for example, the input wave is a pure sinusoid, the output wave will contain a series of harmonics. Some of these harmonics will be of even number, principally the second harmonic, and correspond to a difference of the shapes of the positive and negative half-cycles.

Transposing the connection of one of the 21-type circuits as described above causes one of the grids to receive positive potential from the input wave at the same time that the other grid receives negative potential. This reverses the phase of the even numbered harmonics from one of the 21-type circuits with respect to those from the other, and so eliminates the even numbered harmonics from the output wave of the twin 21-type circuit. This result is similar to that obtained by means of the familiar push-pull arrangement of vacuum tubes used in an amplifier to reduce distortion, but no increase of the number of tubes is required.

The even numbered harmonics from the two 21-type circuits are not annihilated, however, but combine to form an echo which travels toward the speaker, and this echo must not be permitted to become too strong. It would be possible to eliminate such echoes by using the push-pull connection in the amplifier of each 21-type circuit, but this, of course, would double the number of tubes required.

The Twin 21-Type Phantom Group of Repeaters

Three twin 21-type circuits may be connected with the wires of a phantom group so that voice-frequency waves traveling over either side circuit or the phantom may be amplified and low-frequency signals may be passed through the apparatus. This arrangement does not break up the phantom group and requires no phantom repeating coils or compositing apparatus. It cannot be used, of course, at points where it is necessary to separate the side and phantom circuits. A simple diagram of the phantom group of repeaters is shown in Fig. 16. Each 21-type circuit with its own hybrid coil, amplifier, network, etc., is indicated by a small square. A twin 21-type circuit is connected in tandem with each side circuit. The repeater in the side S_1 comprises a line hybrid coil S_1Hy, a series 21-type circuit S_1S and a bridged 21-type circuit S_1B as indicated. The side circuit S_2 is similarly equipped. Repeating coils R_1 and R_2 are shown between the bridged 21-type circuits and the bridge terminals of the line hybrid coils in the side circuits. Taps are provided at the mid-points of the line windings of these coils by which the 21-type circuit PB is bridged across the phantom circuit. While separate transformers are shown in the diagram to provide the connections for the phantom bridged 21-type circuit, they might be omitted if the side circuit bridged 21-type circuits S_1B and S_2B are each so arranged as to provide a tap which is symmetrical with respect to the line wires. This arrangement, however, introduces additional possibilities of unbalance with the resulting noise and crosstalk which are avoided by the use of the coils R_1 and R_2.

The phantom series 21-type circuit PS is coupled effectively in series with the phantom circuit by means of the phantom line hybrid coil PHy. This is a special transformer having eight carefully balanced sections in the line winding and a drop winding to which the 21-type circuit is connected. Two of the line winding sections are connected in series aiding with each line conductor, one on each side of the side circuit line hybrid coil. The sections in series with the several line wires are so poled that they are non-inductive to waves traversing the side circuit, but they are inductive to waves traversing the phantom, thus producing the desired coupling.

The series and bridged phantom 21-type circuits co-act in the phantom circuit to amplify without echoes, the waves traversing the phantom in the same way as the corresponding parts of the side circuits.

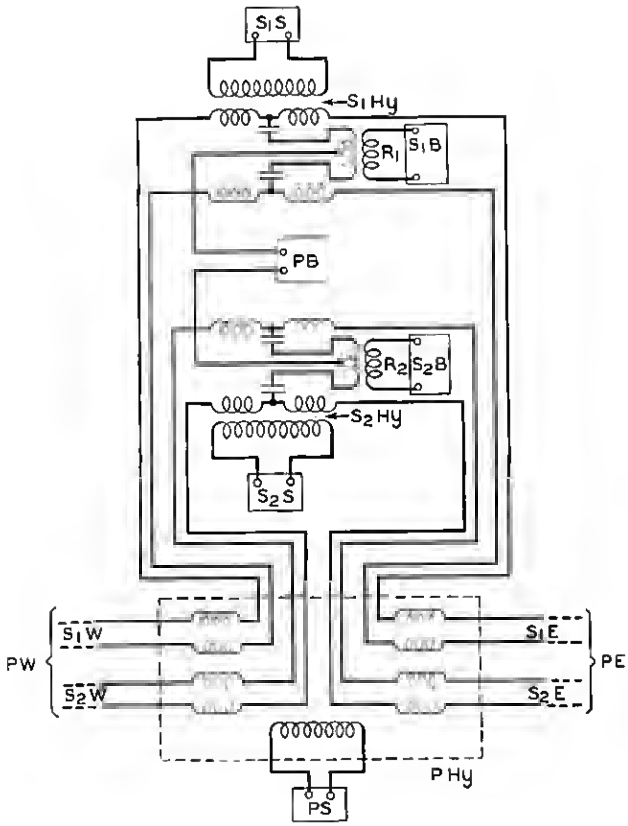

Fig. 16—Twin 21-type phantom group of repeaters.

Field Trials

In order to demonstrate the operativeness of the twin 21-type of repeater and to gain some experience with it, a complete phantom group of repeaters was built, installed at Princeton, N. J., and connected into a phantom group of cable conductors extending from New York to Philadelphia.

This apparatus functioned in a satisfactory manner despite the fact that certain transformers and other parts specially designed for this work were not available, and it was necessary to make use of some equipment designed for other purposes.

Field of Use

Although the results of the field trial were satisfactory, it is not planned to introduce the twin 21-type of repeater into the plant at the present time. It is not yet known whether the features of this type of repeater will prove of advantage as compared with the 21 or 22-type repeaters. However, with the continued increase in the use of repeatered circuits, over which it is desired to transmit d-c. signaling or dial impulses, it is possible that this may be the case and further studies are planned to determine its economic field of use.